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COMMENT

Comment on ‘Excited states in the infinite quantum lens potential: conformal mapping and moment quantization methods’

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Abstract

The problem of quantum lenses has been addressed recently by two papers (Rodriguez *et al* 2003 *J. Phys.: Condens. Matter* **15** 8465, Even and Loualiche 2003 *J. Phys. A: Math. Gen.* **36** 11677). The aim of this comment is to compare the results of both mathematical methods.

The purpose of this comment is to compare and discuss the analytical solutions presented in [1] and [2] for the lens-like quantum dots (QDs).

The two methods are compared with a height to radius ratio equal to $b/a = 1/2$ (figure 1). In the first method the quantum object is described using a conformal mapping of a circular cap and the exact solutions are infinite expansions of an orthonormal complete set of functions. The approximate solution is a truncated series of this expansion [1]. The second method [2] uses a parabolic coordinate system to describe the quantum object and the exact solution is an antisymmetric sum of two separable functions. The symmetry properties of the objects are equivalent in the two descriptions but the two volumes differ by 8.3%. The reduced energy values $E = E_r/E_\infty$ ($E_r =$ real energy, $E_\infty = \hbar^2/2ma^2$) are 60 (62), 83 (88), 108 (116) and 111 (119) for the first (second) method for, respectively, the 1S ($N = 1, m = 0$), 1P ($N = 2, m = 1$), 1D ($N = 3, m = 2$) and 2S ($N = 3, m = 0$) states. Figure 2 represents contour plots of the probability density function for the 1S, 1P, 1D and 2S states calculated from the confluent hypergeometric functions [2]. It is very similar to the results presented in the third column of figure 2 in [1] providing that the 1D and 2S are put in the right order of increasing energy. The two theoretical calculations yield almost the same results. Indeed the QDs profiles are very similar (figure 1) and it would be very difficult to choose between the two on the basis of experimental profile measurements.

It is now possible to have a short discussion on the usefulness of both approaches for the physical description of quantum lenses. In our opinion, the most important requirement for

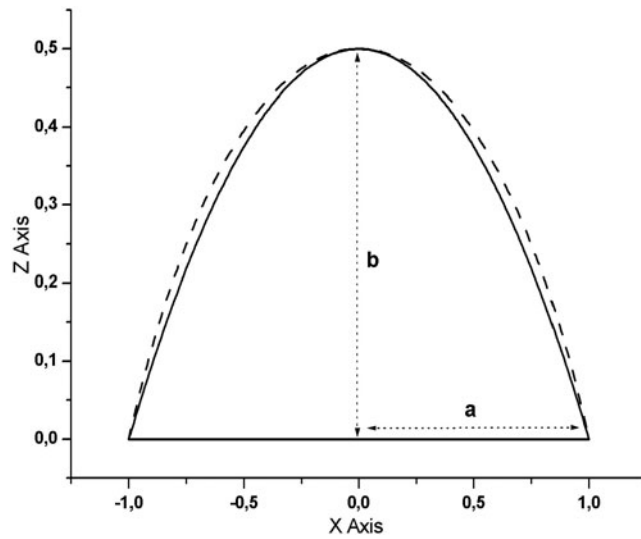


Figure 1. Lens-shaped quantum dot with the height b to radius a ratio equal to $b/a = 1/2$. The broken curve corresponds to the shape defined in [1] and the full curve to the one in [2].

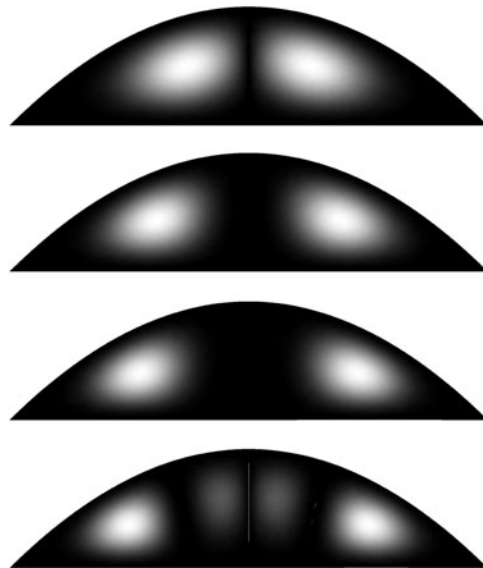


Figure 2. Contour plots of the probability density function for the 1S, 1P, 1D and 2S states calculated from the confluent hypergeometric functions [2].

analytical solutions is simplicity. This is clearly not the case for the method proposed in [1], which is analytical only at the beginning of the calculation. The authors indicate that it is necessary to work with matrices larger than 100×100 in order to obtain an accuracy higher than 10^{-3} for the wavefunctions. We believe that it is as complicated as the common numerical methods such as variational calculations, finite differences or finite elements which, in addition, include finite potential barriers or anisotropic effective masses [3–5]. A second important requirement for analytical solutions is the ability to include easily perturbative calculations

of physical properties useful for the interpretation of experimental studies. Cantele and co-workers have demonstrated in a series of papers [6–8] that separable analytical solutions are particularly well suited for that purpose. Indeed, if we consider, for example, the Stark effect along the vertical axis, it is demonstrated in [2] that it corresponds to a linear effect. A first-order perturbation term may be easily calculated when the perturbation is $-eF(u^2 - v^2)$, where F is the electric field. Simple integrals on u and v are obtained. The introduction of particle–particle interactions is a little bit more difficult. A reliable method consists in using single-particle wavefunctions associated with a trial wavefunction describing the correlated motion [6, 9, 10]. We believe that simple wavefunctions with parabolic coordinates may be a simple starting point for these calculations which correspond to three- or four-fold integrals even for cylindrical coordinates [10]. A third requirement is flexibility. This is met in both approaches by the definition of a dimensionless energy. A drawback of the second approach [2] is, however, that the geometry is fixed whereas it may be changed at will in the first one [1].

We may conclude that neither the first proposed method [1] nor the second one [2] meet all the requirements of completely useful analytical solutions. The geometry may be changed easily with the conformal mapping method. This advantage is, however, lost because cumbersome numerical computations must be performed at the end of the calculation.

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